Model (1) in Algebra

Answer the following questions:

First Question: choose the correct answer of each of the following:

1) If $x+yi=\frac{1-2i}{2+i}$,then $x+y=...$

a) 1 b) – 1 c) 2 d) 3

2) If $1,\omega,\omega^2$ is the cubic roots of one, then $(1+\omega+\omega i)(1+\omega-\omega i)=.....$

a) 4 b) 2 c) 1 d) – 1

3) If ${}^{n}C_{n-2}=36$, then n = ……..

a) 3 b) 9 c) 6 d) 7

4) If the edge length of a regular triangular pyramid equals 3 cm , then its height = …….. cm

a) $\sqrt{2}$ b) 3 c) 6 d) $\sqrt{6}$

5) If the sum of the diagonals lengths of a cube equals = 12 cm. then the area of one face = ……. cm^2.

a) 3 b) $\sqrt{3}$ c) 48 d) 9

6) If a line parallel to each of two intersecting planes then this line is ……

a) perpendicular to their line of intersection
b) parallel to their line of intersection
c) perpendicular to each of them
d) bisect the angle between them.

Second Question:

a) Without expanding. Prove that $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix}=(x+2a)(x-a)^2$

b) MABC is a triangular pyramid, a plane X intersects the edges $\overline{MA},\overline{MB},\overline{MC}$ at D, H and O respectively, where: $\frac{MD}{DA}=\frac{MH}{HB}=\frac{MO}{OC}=\frac{1}{3}$.

Prove that: The plane DHO // the plane ABC.

If $N\in\overline{BC}$, $\overline{MN}$ is drawn to intersect $\overline{HO}$ at E.

Prove that: 1) $\overline{DE}\ //\ \overline{AN}$ 2) AN = 4 DE

Third Question:

a) Using Crammer's method to find the solution set of :

$x+y+z=3$, $x-y+z=1$, $x+y-2z=0$

b) In the triangle ABC, $m(\angle BAC) = 30^o$. $\overline{DB} \perp$ *Plane* ABC. $\overline{DC} \perp \overline{AC}$. DB = 15 cm , AB = 16 cm. find the length of $\overline{DC}$, then find the measure of the dihedral angle $(B - \overleftrightarrow{AC} - D)$.

Fourth Question:

a) If Z is a complex number where $(2-i)\,Z = 4 - \overline{Z}$. Put Z in the trigonometric form, then find the square roots for the number Z in exponential form.

b) X and Y are two planes intersect at $\overleftrightarrow{BC}$, M is a point not belonging to any of them. Draw $\overline{ML} \perp$ the plane X to intersect it at L , $\overline{MN} \perp$ the plane Y to intersect it at N , prove that $\overleftrightarrow{BC} \perp$ the plane MLN.

Fifth Question:

a) In the expansion of $(1-x)^8 (1+x)^8$.

i) Find the coefficient of x^6.

ii) If $\frac{T_6}{T_3} = -128$. Find the value of x.

b) If X is an empty set and $Y = \{(a,b) : a \in X, b \in X, a \neq b\}$, and the number of element of Y equals 72, and $H = \{\{a,b\} : a \in X, b \in X\}$.

Find the number of element of H.

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Model (2) in Algebra

Answer the following questions:

First Question: Complete each of the following:

1) If ${}^{n}P_{r} = {}^{n}C_{r}$, then the value of r is ……..

2) All lines drawn perpendicular to a straight line from a point on it lies in …..

3) ${}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + {}^{n}C_{3} + ... + {}^{n}C_{n}$ = ……

4) The total surface area of a cube is 48 cm^2, then the length of its diagonal = ……

5) $\omega^{98} + \frac{1}{\omega^{98}}$ = ……..

6) If a line is parallel to each of two intersecting planes then this line is …….

Second Question:

a) Find the term free of x in the expansion $\left(x^2 - \frac{1}{3x}\right)^{12}$, then find the ratio between it and the middle term when x = 1.

b) CAB, DAB are two triangles in two different planes. If X, Y, M, Z are mid-points of $\overline{CA}, \overline{CB}, \overline{DA}, \overline{DB}$ respectively. Prove that: $\overleftrightarrow{XY}$ // *plane DAB*, and prove that the figure XYZM is a parallelogram.

Third Question:

c) using crammer method to find the solution set of

$x - 2y + 2z = 2 \qquad 3x + 4z = 10 \qquad 6z - y = 5$

d) X, Y are two planes, where $X \cap Y = \overline{AB}$, the square ABCD is drawn in the plane X, N is the mid point of $\overline{AB}$, M is the mid point of $\overline{CD}$. $\overline{ML} \perp$ the plane Y intersects it at L .

i) prove that $\overline{AB} \perp$ *the plane NML*

ii) If $m(L - \overleftrightarrow{AB} - M) = 60^{o}$. Prove that the plane AML $\perp$ the plane BML.

Fourth Question:

a) If $Z_1 = 2(Sin\frac{\pi}{3} + i\,Cos\frac{\pi}{3})$, $Z_2 = \sqrt{2}(Cos\frac{\pi}{4} - i\,Sin\frac{\pi}{4})$, $Z_3 = 1 + \sqrt{3}\,i$.

Find Z in the exponential form where $Z = \frac{Z_1^{\,5} \times Z_3^{\,3}}{Z_2^{\,4}}$

b) $\overline{MA}$, $\overline{MB}$, $\overline{MC}$ are mutually perpendicular, MA = 8 cm, MB = 7.5 cm, MC = 10 cm. N is the projection of M in the plane ABC.
Prove that:

i) $\overline{BC} \perp$ *the plane* MAN

ii) N is the point of intersection of heights of triangle ABC.

iii) If $\overline{MD}$ is the height of triangle MBC. Find the length of $\overline{MD}, \overline{MN}$

Fifth Question:

a) Prove that: $\frac{1 + 10\omega^2}{1 - 2\omega} + \frac{2 + 17\omega}{2 + 3\omega} = 6$

b) Find the value of k which makes (x – 2) is a factor of the determinant .

$$\begin{vmatrix} x-1 & x+3 & 2 \\ -3 & x+5 & -6 \\ x+3 & 2 & x+k \end{vmatrix}$$